Simulation of the Speed Control for Gas Turbines using Vibrating Reed Regulators

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This paper describes a vibrating reed regulator used to control the speed of a gas turbine driving an alternator. The vibrating reed is excited by a pulsed jet of air at a frequency near to its resonant frequency. The edge of the reed is displaced opposite a fuel pipe, therefore the center section varies as the amplitude of vibrations varies. This amplitude is dependent on the frequency of excitation of the jet and is a function, through the intermediate member to the detector and the acoustic wheel, of the rotational speed of the exit shaft. This regulator is connected to the gas turbine and the entire study of the cycle is studied analogically. However, it must be specified that the study of this control system using a vibrating reed as speed sensor and fuel regulator is a simplified linearized analogical study. The paper presents the theoretical and experimental results concerning the characteristics of the regulator and the performance in a closed loop of the studied system.

Nomenclature

A = reed vibration amplitude
a = $(EI/\rho_1 S_1)^{1/2}$, speed of sound in the reed
b = width of the reed
D = diameter of the fuel discharge pipe
E = young's modulus of the reed metal
e = thickness of the reed
e^* = e/D, reduced thickness
F = force applied to the extremity of the reed
F_M = amplitude of the force
f = frequency of the signal exciting the reed
f_R = resonant frequency of the reed
G = $-\Delta Q_c/\Delta N$, static gain of the regulator associated with the reduction gear
G_1 = $\Delta N/\Delta Q_c$, static gain of the turbine
G_2 = $-\Delta N/\Delta W$, static gain of the turbine
G^* = $-\Delta Q_c/\Delta f$, static gain of the regulator
I = moment of inertia of the cross section of the reed
J = moment of inertia of the rotating parts of the turbine
L = length of the reed
m = number of castellations on the acoustic wheel
N = speed of rotation of the turbine
n = speed of rotation of the acoustic wheel
P = atmospheric pressure
ΔP = pressure upstream of the discharge pipe
p = Laplace's variable
Q = rate of flow of fuel discharged by the pipe
Q_N = nominal rate of flow of fuel given by the fuel flow regulator
Q_c = rate of flow of fuel injected
S = section of passage of fuel in the pipe-reed system
S^* = S/S_M, reduced section
S_M = $\pi D^2/4$, maximum section of passage of fuel
S_m = $\pi D \cdot \delta$, minimum section of passage of fuel
S_1 = cross section of the reed
T = atmospheric temperature
t = time
W = power delivered by the turbine
x^* = $e^* - 2y^*$,
$Y(p)$ = transfer function of the displacement y of the reed
y = lateral displacement of the reed
y^* = y/D, reduced displacement of the reed
yM = static displacement corresponding to the force F_M
α = angle
β_i = $(\omega_i/a)^{1/2}$,
γ = reduction gear ratio
Δ = indicates an increase with respect to the nominal value
δ = distance between the fuel pipe and the reed
δ^* = $4\delta/D$, reduced distance
ϵ = static error
θ = T/T_s,
ξ = damping coefficient of the speed controller
ξ_1 = damping coefficient of the reed
$\tilde{\omega}$ = P/P_s,
ρ = specific gravity of fuel
ρ_1 = specific gravity of the reed metal material
σ_L = limit stress
σ_M = maximum stress
τ_1 = time constant of controller
τ_2, τ_3 = time constants of turbine
χ_1, β_2 = characteristic constants of the reed material
ω = angular frequency
ω_i = fundamental frequency corresponding to the first mode of vibration of the reed

Subscripts

R = corresponds to resonance
0 = corresponds to the nominal working of the machine N_0 = 50000 rpm; W_0 = 10 kw
s = corresponds to the standard conditions of pressure and temperature (P_s = 1013 mbars; T_s = 288 K)

Superscripts

— = time average value
′,″ = time derivatures

I. Introduction

THE initial idea, which lead to the study of a speed regulator of the vibrating reed type for a gas turbine, was to provide a permanent assembly capable of functioning by means of the source of power which consists of the gases under pressure from the compression chamber. The problem consists therefore of maintaining constant rotational speed of a gas turbine driving an alternator, whatever the electric power supplied, and taking into account the important variations of atmospheric parameters due to changes in altitude, acting on the quantity of fuel injected into the combustion chamber.

In fact, the prototype realized is not the first step into the field of speed regulation of turbines or motors. One can cite the solution of the on-off control type of Katz and Iseman[1] and the delay line speed pickups of Tonegawa and Kawasaki,[2] Law,[3,4] Noel,[5] the Plessey Co.,[6] Wolf,[7] and Vamvakoussis.[8]

Several techniques of demodulation have been used.[9-14] If a comparison is made of all these developments, we will see that the solution which consists of using the built-in reeds as an oscillating system, is the only one which might guarantee high stability output characteristics when the temperature varies. In the case of a turbine, the air sampled at the compressor outlet undergoes large variations of

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temperature caused by, on the one hand, the variation of ambient temperature and on the other, by the variation of the rate of compression when the power delivered varies. A simple entirely fluidic system is not susceptible of assuring the required performance.

II. Principle and Description of the Vibrating Reed Regulator

One of the goals has been to supersede the control valve generally used to control the fuel supply. The device studied (Fig. 1) is composed of a bank of injectors supplying the combustion chamber with a rate of fuel flow Q_c. Upstream there is a flow regulator, independent of the device studied, which gives a constant fuel flow Q_n. The discharge gate connected in parallel with the injector is composed of a pipe before which a reed vibrates. This reed, built in at one end, is excited at the other end by the square wave signal given by a speed detector which interrupts a jet with a disk. The jet is given by the compressor of the gas turbine. When the frequency of excitation is near to the resonant frequency of the reed, it vibrates with an amplitude whose variation with frequency bears an inverse relationship with its damping.

The fuel discharge pipe is placed opposite the edge of the reed (Fig. 2). If the distance from the reed is small enough, the displacement y of the reed gives a variation in the effective exit area, and therefore the rate of fuel flow Q which it allows through there is the relation: $Q_N = Q_c + Q$.

When the reed vibrates with an amplitude A, the average rate of flow $\bar{Q}$ increases when A increases. If $f < f_R$, when the speed increases, A and Q increase and Q_c diminishes which tends to decrease the rotational speed. Since it is not linear, such a system does not behave in practice like a proportional regulator. It is not possible to annul the static error, and one has to acknowledge tolerance in the measurement of the speed if one wishes to assure stability. Consequently, it is only possible to act on the gain of the regulator, that is to say, the slope of the characteristic $\partial \bar{Q}_c/\partial N$ in the neighborhood of the working point. And so the knowledge of the characteristics of the machine let us determine the value of the gain of a proportional system.

In the study of this device, the dynamics of the fuel system and the pneumatic system are neglected. It is however possible that for rapid variations of the speed the dynamic of the fuel system has a significant effect on the system. At least, all the characteristics are linearized in the neighborhood of the working point. The simulated studies of the chain will show whether or not its structure has been too idealized.

III. Determination of Optimal Gain

The turbine whose regulation we are studying is mechanically attached to a compressor driving an alternator. The rotational speed N is function of several parameters: $N = N(Q_c, W, P, T)$.

The relationship is simplified by dimensional considerations, and one can interpose three reduced parameters[15,16]

$$N_s = N/\theta^{1/2} \quad W_s = W/(\bar{\omega} \times \theta^{1/2}) \quad Q_{cs} = Q_c/(\bar{\omega} \times \theta^{1/2})$$

where $\bar{\omega} = P/P_s$ and $\theta = T/T_s$, P_s and T_s being, respectively, standard pressure and temperature (P_s = 1013 mbars, T_s = 288 K). Thus we can write $N_s = N_s(Q_{cs}, W_s)$.

This relationship holds in steady state. A study of the parameters of the turbine shows that the transfer function of the former for small variations can be put in the form

$$\Delta N(p) = \frac{G_1 \times \Delta Q_c - G_2 \times \Delta W}{(1 + \tau_2 \times p)(1 + \tau_3 \times p)}$$

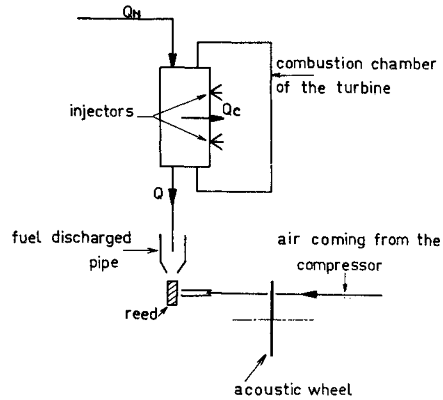

Fig. 1 Scheme of the fuel circuit and regulator.

One shows /16/ that $\tau_2 = JN_oG_2$ (N_o: nominal speed) and τ_3 a time constant attributed to the thermic inertia of the combustion chamber. These times constants are of the form $\tau_s = \tau\bar{\omega}/\theta^{1/2}$.

The relationship above can therefore be put in the form

$$\Delta N(p) = \frac{1}{\bar{\omega}} \times \frac{G_{1s} \times \Delta Q_c(p) - G_{2s} \times \Delta W(p)}{(1 + (\theta^{1/2}/\bar{\omega}) \times \tau_{2s} \times p)(1 + (\theta^{1/2}/\bar{\omega}) \times \tau_{3s} \times p)} \quad (1)$$

where G_{1s}, G_{2s}, τ_{2s}, τ_{3s} are constants independent of atmospheric conditions.

This calculation of optimal gain G of the proportional controller can be made if one knows these four parameters and if the range of the variation of $\bar{\omega}$ and θ are fixed. Suppose the regulator does not introduce either delay or dead time. By definition we have

$$G = -\left|\frac{\Delta Q_c(p)}{\Delta N(p)}\right|$$

Therefore the expression $N(W)$ becomes

$$\frac{\Delta N}{\Delta W} = -\frac{G_2}{1 + GG_1} \times \frac{1}{1 + \frac{\tau_2 + \tau_3}{1 + GG_1}p + \frac{\tau_2\tau_3}{1 + GG_1}p^2}$$

The static error and damping of the system are deduced from

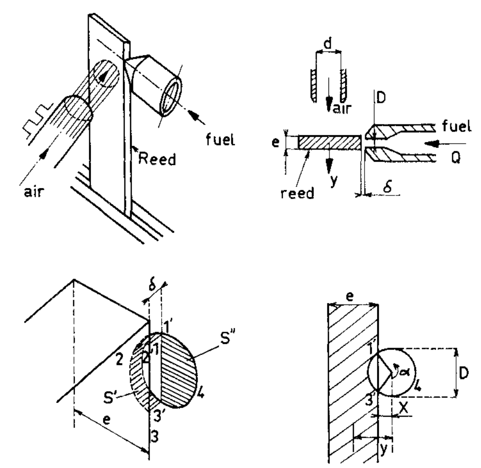

Fig. 2 General scheme and details of the vibrating reed and discharge pipe.

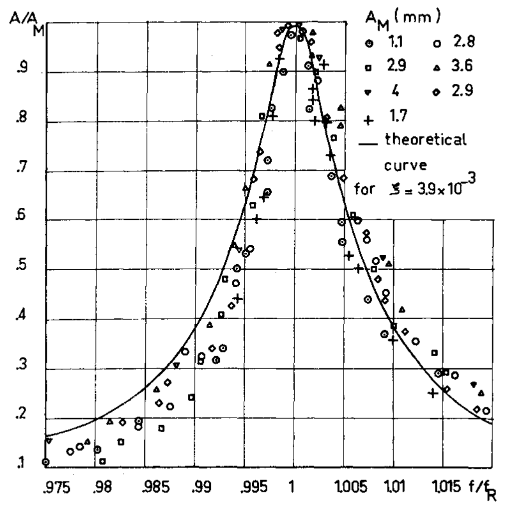

Fig. 3 Relative amplitude vs the relative frequency of the vibrations of a built-in reed.

$$\epsilon = -\frac{G_2}{1 + GG_1} = -\frac{1}{\bar{\omega}} \times \frac{G_{2s}}{1 + G_s \times G_{1s}}$$

$$\xi = \frac{\tau_2 + \tau_3}{2[(1 + GG_1)\tau_2\tau_3]^{1/2}} = \frac{\tau_{2s} + \tau_{3s}}{2[(1 + G_s \times G_{1s})\tau_{2s} \times \tau_{3s}]^{1/2}}$$

It can be seen that the damping does not depend on either $\bar{\omega}$ or θ. The values of the parameters for the regulated turbine are

$G_{1s} = 10.7$ rps/1/h $G_{2s} = 5.3$ rps/kw $N_{so} = 833$ rps
$W_{so} = 10$ kw $Q_{cso} = 41$ 1/h $\tau_{2s} = 0.7$ sec $\tau_{3s} = 0.1$ sec

By choosing $G_s = 0.8$ 1/h/rps, the values of ξ and ϵ are as for standard temperature and pressure conditions: $\xi = 0.45$; $\epsilon = 0.55$.

First, suppose that atmospheric conditions are fixed at the standard values. G_s being determined, one has to ascertain the gain $G^*_s = (\Delta Q_c/\Delta f)_s$. The acoustic wheel has m castellations and is mounted on a shaft rotating with speed n. One can write, calling $\gamma = N/n$ the reduction gear ratio: $G^* = G \cdot \gamma/m$ γ is constant and equal to 6.25 while one is free to choose the value of m. $G^*_s = 5/m$ 1/h/Hz. It is necessary to determine the best value of m. Several reminders on the vibrations of built-in reeds allow us to be precise on this point.

IV. Notes on the Vibrations of Built in Reeds

Figure 3 shows the shape of the response curve of a reed built-in at one end. Consider the force $F = F_M \sin \omega t$ applied at the free end. Let y_M be the static deflection of the reed corresponding to the force F_M. For a reed of constant cross section one has

$$y_M = (F_M \times L^3)/3EI \tag{2}$$

The forced vibration of the free end of the reed is a linear superposition of the response of the reed at each vibration mode.[17] The deflection, without energy losses, is of the form

$$y = \frac{4F_M}{\rho_1 \times S_1 L}\left[\sum_{i=1}^{\infty} \frac{1}{\omega_i^2 - \omega^2}\right] \times \sin \omega t$$

ω_i is the undamped natural frequency of the ith mode. It is possible to approximate the reed in the neighborhood of the first resonant frequency to a second-order system.[16,18]

One can see that the resonant frequencies ω_i corresponding to the different vibration modes are not in a geometric progression. In particular ω_i are not exact multiples of ω_1 whose value is

$$\omega_1 = 1.015 \times (e/L) \times (E/\rho_1)^{1/2} \tag{3}$$

This signifies, among other things, that if the reed is excited by a periodic force of frequency ω near to ω_1, the frequencies of the different harmonics are very probably different to the resonant frequencies ω_i.

Therefore the response of the conservative system is

$$\frac{y}{y_M} = \frac{0.97}{1 - (\omega/\omega_1)^2} \times \sin \omega t$$

In fact, energy is dissipated, in particular, from the built-in surfaces, which leads to the introduction of a damping coefficient ξ_1 and to the adoption of the following transfer function for the identification of the reed:

$$Y(p) = \frac{0.97}{1 + 2\xi_1 \times \frac{p}{\omega_1} + \frac{p^2}{\omega_1^2}}$$

In fact we are interested in the response $A(\omega)$ or $A(f)$ of the system. It is possible to show that the assimilation of $A(f)$ to a first-order system whose time constant would be

$$\tau_1 = \frac{1}{\xi_1\omega_1} = \frac{1}{2\pi\xi_1 f_R} \tag{4}$$

So, to have a good response time, f_R must be as large as possible. As the frequency f of the signals is near to f_R this means that the number m should be chosen large. However, other considerations are inconsistent with this.

Let σ_M be the maximum value of the stress for a vibration of amplitude A. For a reed of rectangular cross section, it can be shown that it is equal to $\sigma_M = 1.75$ AE e/L^2. If σ_L is the limit stress, it is deduced from Eq. (3) that the maximum amplitude A_M must be such that

$$A_M < \frac{0.58}{\omega_1} \times \frac{\sigma_L}{(\rho_1 E)^{1/2}} = \frac{\chi_1}{\omega_1}\left(\text{with } \chi_1 = \frac{0.58 \times \sigma_L}{(\rho_1 E)^{1/2}}\right) \tag{5}$$

The relationship (2) becomes with (3)

$$\frac{yM}{F_M} = \frac{4.08}{b(e\omega_1)^{3/2}} \times \frac{1}{\chi_2} \quad \text{with } \chi_2 = (\rho_1{}^3E)^{1/4} \tag{6}$$

It can be seen as a result that the mode ω_1 is increased and therefore that f_R, the smaller the maximum attainable amplitude becomes [Eq. (5)] and the smaller the coefficient of safety.

The greater ω_1 is, the greater the force F_M has to be for a given deflection y_M [Eq. (6)]. This signifies for one that the energy used by the regulator increases as $\omega_1{}^{3/2}$. The adopted value of m must be a compromise ensuring sufficient speed and at the same time assuring correct safety and economy of energy. For these reasons m was taken equal to 1. Therefore the gain G^* becomes $G^* = 5$ l/h/Hz.

In order to give some idea of the speed of the detector, we will state precisely the values of ξ_1 and ω_1 for the application considered. Frequency ω_1 is equal to 835 Hz. The value of ξ_1 depends on the way in which the reed is built in as much as on the mass on which the reed is fixed. It is possible with the help of experimental curves of the amplitude to calculate the value of ξ_1. For steel 35 NCD 16, and aluminium alloy AU 2 GN, ξ_1 is equal to 4×10^{-3}.[16] However, the presence of fuel increases the dissipation of energy and ξ_1 increases slightly and becomes 7×10^{-3}. The time constant τ_1 will be in this case equal to: $\tau_1 = 0.17$ sec.

V. Analytic and Experimental Study of Characteristic *Q(y)*

The simulated study of the functioning of the reed requires knowledge of the function $Q(y, \delta, D, e, \Delta P)$. The area of passage of fluid is defined by

$$S = Q/c\left(\frac{2\Delta P}{\rho}\right)^{1/2}$$

where c is the rate of discharge coefficient for the pipe by itself. Therefore the parameter S becomes a function of entirely geometric variables: $S = S(y, \delta, D, e)$. We assume in all that follows that the reed covers the pipe ($e > D$). S being a geometrically complex surface, there is no question of being able to know it directly. However, it is sufficient to be able to determine an equivalent parameter having the dimension of a surface and verifying the formula which defines it. To measure S we have therefore proceeded in the following manner: one calculates the coefficient of rate of discharge of the tube by itself by several measurements of Q and ΔP; the value of c being known, one can then, for different values of y, δ, D, e, measure Q and ΔP to determine the value of S.

A. Effect of the Pipe-Reed Distance

It is easy to find two limiting values of S when δ varies. If δ is large with respect to D, S will be equal to S_M such that: $S_M = \pi D^2/4$. If δ is small with respect to D, the section will be that of a circular ring: $S_m = \pi D\delta$.

If we let: $S^* = S/S_M$ and $\delta^* = S_m/S_M = 4\,\delta/D$; the curves $S^*(\delta^*)$ can be drawn with adimensional values. The experimental tests show that linear approximation $S^* = \delta^*$ is good if $\delta < D/16$, or $\delta^* < 0.25$.

B. Determination of the Equivalent Section $S(y)$

Suppose that the parameters D, e, δ are fixed, so that $e > D$ and $\delta^* < 0.25$. It is possible to find an analytical approximation of $S(y)$. S can, in fact, be considered (Fig. 2) equal to the sum of the two surfaces: the annular part S' and the sector S'' of the circle uncovered by the reed

$$S = S' + S'' = \delta(2\pi - \alpha)(D/2 + (\alpha - \sin\alpha)(D^2/8)$$

By putting $x = (D/2)\cos(\alpha/2)$ and relating the geometric parameters to D in such a way as to make them adimensional, all the equations become

$$S^* = \delta^* + \frac{1 - \delta^*}{\pi}\text{arc}\cos x^* - \frac{x^*}{\pi}(1 - x^{*2})^{1/2}$$

with

$$x^* = e^* - 2y^*,\ e^* = e/D,\ dy^* = y/D$$

Figure 4 shows that the experimental results correlate with the theoretical curves. However, the relationship above is not easy to simulate analogically.

In order to make this simulation easier, we have compared the curve $S^*(y^*)$ to a curve formed by three straight segments, the two limits being defined by S^* = constant after the limit has been attained. This approximate curve is defined by

$$\begin{array}{ll} 0 \le |y^*| \le \dfrac{e^* - \pi/4}{2} & S^* = \delta^* \\ \dfrac{e^* - \pi/4}{2} < |y^*| < \dfrac{e^* + \pi/4}{2} & S^* = \dfrac{1 + \delta^*}{2} \\ & \quad - \dfrac{2}{\pi}(1 - \delta^*)(e^* - 2|y^*|) \quad (7) \\ |y^*| \ge \dfrac{e^* + \pi/4}{2} & S^* = 1 \end{array}$$

S^* can be deduced from knowledge of S. Figure 4 shows that the linear approximation does not differ too much from experimental results.

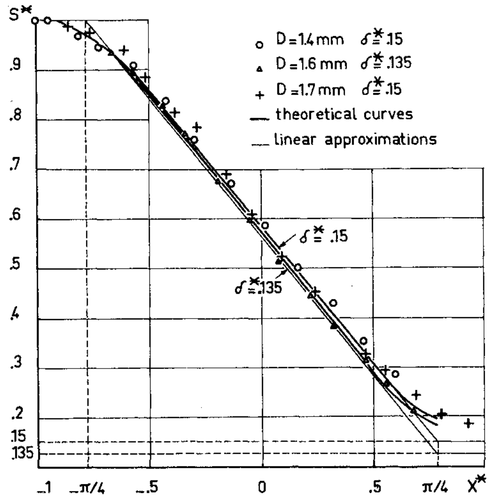

Fig. 4 Variation of the section S^* vs X^*.

VI. Several Observations on the Analogical Simulation

The analogical circuit of the turbine and its regulator integrated with the fuel supply circuit has been achieved with the help operational amplifiers. It is composed of four principal parts: a) simulation of the turbine by its identification with the relation $\Delta N(\Delta Q_c, \Delta W)$, b) simulation of the fuel supply which links ΔQ_c with S, c) simulation of the reed corresponding to section S with an exciting frequency f, and d) simulation of the accoustic wheel joining parameter f to the speed variation ΔN about a nominal point. Figure 5 shows the scheme of the simulator.

A. Simulation of the Turbine

The identification of the parameters of the gas turbine shows that it transfer function is given by formula *(I)*, and variables ΔN, ΔW, ΔQ_c represent the variations about a nominal point. Let N_o = 50,000 rpm, W_o = 10 kw, Q_{cso} = 41 l/h. Therefore the simulation circuit will have in series, a differentiator which calculates the function $G_{1s} \times \Delta Q_c - G_{2s} \times \Delta W$, an amplifier with a gain control which al-

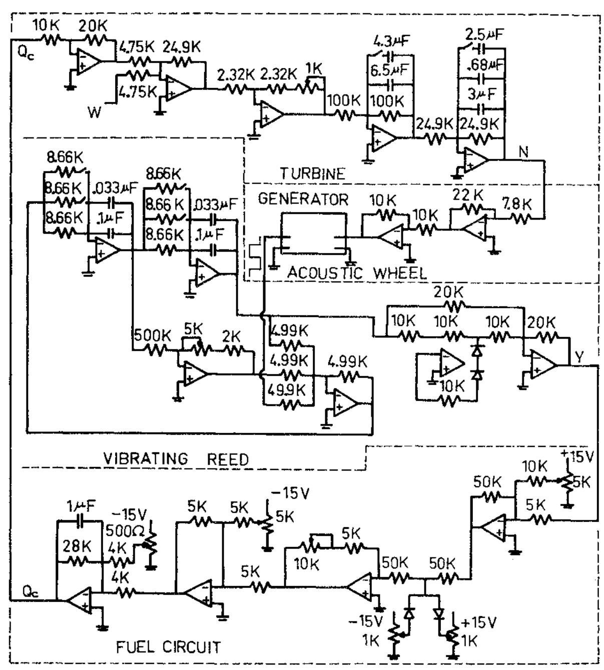

Fig. 5 Scheme of the simulator.

lows the adjustment of the factor $1/\tilde{\omega}$ which corresponds to variation in altitude, and two active low-pass filters with a time constant, each provided with a control which permits the variation of the factor $\theta^{1/2}/\tilde{\omega}$.

B. Simulation of the Accoustic Wheel

A continuous voltage must be transformed into a signal of fixed amplitude and of frequency proportional to the speed of rotation. For this, a signal generator with frequency variable by a constant voltage was used.

As the frequency of the exit signal is equal to f, one can adjust the variable gain circuit so that one obtains $\Delta f = (m/\gamma) \times \Delta N$. The generator is regulated in such a way that when $\Delta N = 0$, $f = f_o = (m/\gamma)N_o$. The choice of the amplitude of the signals is such that when $\Delta W = 0$, ΔN is equal to 0, which corresponds to a centering of the proportional band.

C. Simulation of the Reed

As the reed is a second-order system with little damping, the analogical circuit solves an equation of the type $y''/\omega_1^2 = -(y + \xi_1 y'/\omega_1 - F_M \sin \omega t)$ where $F_M \sin \omega t$ is the signal coming from the signal generator.

Therefore the circuit consists of an adder and two integrators whose integration constants are identical. A variable gain circuit enables the damping ξ_1 to be adjusted. The exit signal y which represents the deflection of the reed is then rectified to give $|y|$ since it is the last parameter which interferes in the calculation of the section of the fuel flow. The section of fuel flow S can be obtained from Eq. (7), joining S^* to δ^* and $|y^*|$. The characteristic consists of three straight segments. The analogical approximation is made with the aid of diode circuits.

D. Simulation of the Fuel Circuit

We have seen that $Q_N = Q_c + Q$ and for nominal working, $Q_N = Q_{co} + Q_o$; so we have $\Delta Q_c = -\Delta Q$. The determination of $\Delta Q(S)$ leads to that of $\Delta Q_c(S)$.

One can show[16] that with the fuel used it is possible to linearize the characteristic $\Delta Q_c(S)$ and that one can adopt a new relation: $Q_c = 122 - 70\,S$ where Q_c is expressed in l/h and S in mm^2. The value of ΔQ_c will naturally be a function of the nominal rate of flow Q_{co} which depends on the altitude $\Delta Q_c = 122 - Q_{co} - 70\,S$.

VII. Study of the Static Characteristic $\Delta Q_c(f)$

The determination of the gain G^* in the neighborhood of the working point can be made with the aid of the static characteristic $\Delta \bar{Q}_c(f)$. Figure 6 gives this characteristic for two values of f_o/f_R ($f_o/f_R = 0.95$ and 0.97). This characteristic has been plotted with the aid of an analogical circuit by slowly varying the frequency and filtering the signal ΔQ_c in such a way as to obtain the average value. The average rate of flow $\Delta \bar{Q}_c$ and the continuous voltage proportional of f are recorded on a plotter. The curve has identical shapes. The injected rate of flow decreases slightly at first when the frequency is raised, then the slope of the curve is accentuated, and $\Delta \bar{Q}_c$ cancels itself out for the nominal working point and becomes minimum when the resonant frequency is attained.

The curves can be characterized by three important parameters: by the slope in the neighborhood of the nominal point on which depends the stability of the control, and by the variations of maximum and minimum flow rates which determine the extreme flow rates which can be injected into the machine. It is, in fact, necessary to respect two conditions: first, not to inject too large a flow rate so that the temperature on the turbine blades does not rise, and second, not to inject too small a flow rate which could give rise to extinction of the flame.

To obtain the ideal adjustment, the suitable values of the following parameters Q_{co}, D, e, δ, f_R must be chosen. The nominal flow rate Q_{co} at mean load is only dependant on the altitude, for the trials it is fixed at the value it has under conditions of standard temperature and pressure; that is, 41 l/h. The diameter D is given by S_M the maximum value of the section S. The thickness e of the reed must be greater than $D(e^* \geq 1)$ and δ must be chosen so that $\delta^* < 0.25$. Figure 6 gives also the characteristics $Q(f)$ for different values of δ^*.

The gain in the neighborhood of the working point is not sensitively influenced by parameter δ^*. But the maximum variation of ΔQ_c depends strongly on δ^* and the difference is all the greater the farther one moves from the nominal frequency. This corresponds to the position of rest or to reed oscillation of very small amplitude. In this configuration, the mean section of passage of fuel is practically equal to the annular section $S_m = \pi\ D\ \delta$, a relationship which implies the hypothesis of linearization that Q_c may be proportional to δ. The distance from pipe to reed can therefore be used to regulate the maximum injected flow rate.

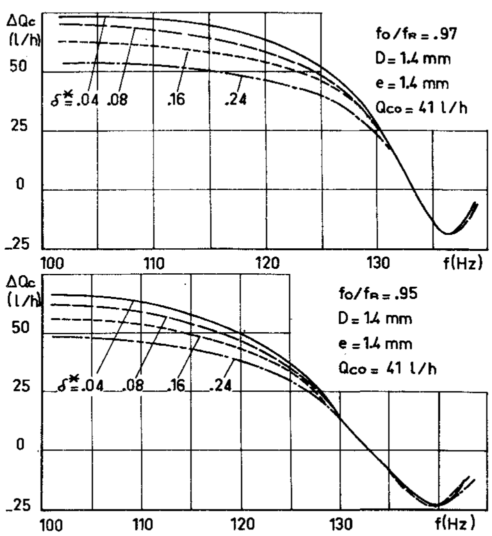

Fig. 6 Variation of the injected flow Q_c with the frequency of the vibrations of the reed.

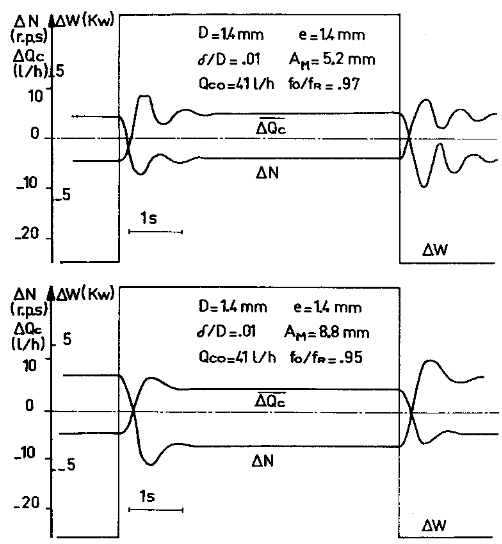

Fig. 7 Recording of ΔN and ΔQ_c for a variation of the load ΔW.

If the value f_o of the nominal frequency is given, it is necessary to know what the value f_R of the resonant frequency will be. It f_R moves away from f_o, this signifies that on the amplitude frequency curve of Fig. 3, the working point is displaced towards the bottom such that the gradient in the neighborhood of this point diminishes. Then when f_o/f_R diminishes, the gain diminishes.

This result is evident on Fig. 6 where f_o/f_R has the respective values of 0.95 and 0.97. Furthermore, if f_o/f_R is decreased and if the exciting force stayed the same, the amplitude would diminish and the injected rate of flow would increase. It is therefore necessary when f_o/f_R diminishes to increase the amplitude of the exciting force in such a way as to obtain the same rate of flow Q_c at the nominal state. The diminution of f_o/f_R also leads to a diminution of the minimum injected flow rate.

VII. Experimental Study of Speed Regulation

The two values adopted for f_o/f_R lead, in the neighborhood of the working point, to the following gains

$$f_o/f_R = 0.97 \qquad G^* = 7.8 \text{ l/h/Hz}$$

$$f_o/f_R = 0.95 \qquad G^* = 4.4 \text{ l/h/Hz}$$

The dynamic study of a closed-loop system has been made for these two cases. The maximum load is 20 kw. The normal rate of power is 10 kw and this working is taken as origin for ΔQ_c, ΔN, and also ΔW which then varies between -10 kw and $+10$ kw. For the two values of f_o/f_R, Fig. 7 shows the variations of ΔN and ΔQ_c when ΔW varies from -10 kw to $+10$ kw and vice versa.

On the curve of Fig. 7 corresponding to a gain of 4.4 l/h/Hz, there does not appear to be a perceptible difference between a positive step and a negative step. The system is correctly damped and the static error reaches $\pm 0.72\%$ for a variation of maximum load ($\Delta W = \pm 10$ kw). It is possible to reduce the static error to the detriment of stability by increasing gain; so, for a gain of 7.8 l/h/Hz (Fig. 7), the error in speed is less than $\pm 0.50\%$ but the presence of oscillations shows that the damping has diminished.

The response time is therefore increased and also the oscillations do not represent the same form depending on whether ΔW is positive or negative. In fact, when ΔW is positive, the variation of speed is negative and the frequency of vibration of the reed is moved away from the resonant frequency. When ΔW is negative, the working frequency of the reed reapproaches the resonant frequency, and the system is in fact much less damped; consequently the amplitudes of vibration of ΔN and of ΔQ_c are more important when ΔW is negative. In fact, the nonlinear response of the system appears clearly in this case.

The case for which $G^* = 4.4$ l/h/Hz is therefore much more satisfying, particularly from the point of view of stability and response time. The static error determined under the most unfavorable load conditions has an acceptable value. However, the variations of exterior conditions by a change in altitude can give it much more important values.

IX. Influence of Altitude on Static Error

Assume that the load does not change ($\Delta W = 0$) and that the rotational speed is always practically equal to the nominal speed ($\Delta N = 0$). The point in question is the determination of ΔQ_c the variation in the injected flow rate as a function of parameters $\Delta\bar{\omega}$ and $\Delta\theta$. Use the system of equations

$$N_s = N/\theta^{1/2} \qquad W_s = W/\bar{\omega} \times \theta^{1/2}$$

$$Q_{cs} = Q_c/\bar{\omega} \times \theta^{1/2} \qquad N_s = N_s(W_s, Q_{cs})$$

Differentiate and write $dW = 0$ and $dN = 0$. After linearization,

$$\Delta Q_c = \left(Q_{cso} - \frac{G_{2s}}{G_{1s}} \times W_{so} \right) \Delta\bar{\omega} - \left(\frac{G_{2s}}{G_{1s}} \times W_{so} + \frac{N_{so}}{G_{1s}} - Q_{cso} \right) \times \frac{\Delta\theta}{2} \quad (8)$$

The expression becomes, with numerical values, $\Delta Q_c/Q_{co} = 0.9\,\Delta\bar{\omega} - 0.5\Delta\theta$.

The machine must be able to function at an altitude between 0 and 3000 meters, on the ground the existence of standard conditions ($T = 288$ K, $P = 1013$ mbar) is assumed and at 3000 meters, the conditions are $T = 268$ K and $P = 710$ mbar. These conditions give the following variations of the parameters: $\Delta\theta = -0.07$ and $\Delta\bar{\omega} = -0.3$.

The relative variation in the injected flow rate will therefore be of the order of 23%. Thus for the machine under study consuming 41 l/h on the ground, an increase in altitude of 3000 meters diminishes the injected flow rate to no more than 31.5 l/h.

If the machine is equipped with a proportional controller such that the gain is equal to 4.4 l/h/Hz, the increase in speed which will result from such a change is of the order of 2%. Therefore, it appears that the variations of altitude or more generally changes in ambient conditions lead to static errors of the same order of magnitude as the variations in load.

X. Conclusions

The proportional controller studied has allowed us to obtain interesting dynamic performances on a gas turbine; in particular from the point of view of response time. However, the static error is not zero and small variations of speed must be tolerated about the working point when the load or the atmospheric conditions vary. If not, the proportional controller is insufficient and it is necessary to envisage the introduction of an integral stage in the controller. But this control device has several advantages of great practical interest: its simplicity of development leading to low production costs, the independence which it procures for the turbine by using gas from the compressor and, lastly, the absence of friction between mechanical parts, the only moving piece being the reed which vibrates about its equilibrium position.

From many aspects it is moreover this last point which seems the most interesting, for generally, in practice, servovalves of high production cost are used whose characteristics do not always respond to the requirement of the users.

References

[1]Katz, S. and Iseman, J. M., "Angular Speed Control with a Bistable Fluid Amplifier," *Control Engineering*, Vol. 7, 1963, pp. 133–135.

[2]Tonegawa, K., "Fluidic Governor for Diesel Engine," *Journal of the Japanese Association of Automatic Control Engineers*, Vol. 13, No. 1, 1969.

[3]Law, A. C., "Pneumonic Control of Marine Engines," *Fluid Power International*, Vol. 1, 1968, pp. 19–24.

[4]Law, A. C., "Fluidic Control of Marine Diesel Engines," *Fluidic International*, Vol. 1, No. 2, 1968, pp. 24–29.

[5]Noel, P., "Application de la Fluidique aux Turbomachines et bancs d'Essais," Automatisation Fluidique et Pneumatique," *Journees d'Orsay*, Vol. X, 1970, pp. 171–195.

[6]Plessey Ltd., "Improvements in or relative to Control Systems, Particularly for Gas Turbine Engine Inlet Guide Vanes," British Patent 1 225 119, 1970.

[7]Wolff, "Speed sensing method and apparatus," British Patent Spec. 1 227 982, 1970.

[8]Vamvakoussis, G., "Etude et Réalisation d'un Détecteur Fluidique de Vitesse Angulaire. Application à la Régulation de Vitesse," Thèse de Docteur Ingénieur, 1971, Univ. de Toulouse, Toulouse, France.

[9]Reilly, R. J., "All Fluid Devices and their Application to Gas

Turbine Controls," Society of Automotive Engineers, Mid-Year Meeting, May 1965.

[10]Reilly, R. J., "A Survey of Fluidic Components and their Application to Gas Turbine Control," AGARDograph 120, 1969, pp. 379–439.

[11]Colson, J. R. and Dexter, E. M., "Application of Pure Fluid Techniques to a Speed Control," *Proceedings of the Second Symposium on Fluid Amplitude, Harry Diamond Laboratories,* Vol. 2, 1964, pp. 405–429.

[12]Boothe, W. A., "Feasibility Study of a Fluid Amplifier Steam Turbine Speed Control," *Proceedings of the Second Symposium on Fluid Amplitude, Harry Diamond Laboratories,* Vol. II, 1964, pp. 377–403.

[13]Boothe, W. A., Ringwall, C. G., and Shinn, J. N., "New Fluid Amplifier Techniques for Speed Controls," *Proceedings of the SAE Aerospace Fluid Power Systems and Equipment Conference,* Vol. 5, 1965, also *Joint Automatic Control Conference,* Vol. VI, 1965.

[14]Boothe, W. A., Ringwall, C. G., Shinn, J. N., and Kelley, L. R., "A Fluid Amplifier Technique for Speed Governing using Carrier Frequency Modulation," presented as paper 31G at the 3rd International Federation of Automatic Control Congress, 1966.

[15]Comolet, R., *Mécanique Expérimentale des Fluides,* Tome II, Masson Editeur.

[16]Sanjurjo, L., "Etude et Réalisation d'un Régulateur Proportionnel Fluidique. Application au Contrôle d'une Turbine à Gaz," Thèse de Docteur de 3e cycle, 1972, Univ. de Toulouse, Toulouse, France.

[17]Timoshenko, S., *Vibration Problems in Engineering,* Van Nostrand, Princeton, N.J., 1955.

[18]Dat, J., Fabre, J., and Sanjurjo, L., "A Vibrating Reed Control Valve and its Applications to Speed Control," Congrès Jurema, Zagreb, April 1973; also *Fluidic Quarterly,* Vol. 6, No. 1, Jan. 1974, pp. 1–19.